

1.5. Homework ♣ ♣

This week concerns preparatory work for a theme on a re-enactment of a sea voyage from Tahiti to Gisborne in New Zealand. We shall go on to explore trigonometry, units conversion, vectors, differential quantities, use of computer packages (Excel or similar). These are related to engineering drawings, surveying, navigation (self driving vehicles), robotics, dynamics, and Fluid Mechanics.

Homework: New locations ♣ ♣

(A) Complete the multiple choice questions on the next page. (B) Visit <https://demonstrations.wolfram.com/DeterminingLatitudeFromMaximumSolarAltitude/>. Try out locations below the Tropic of Capricorn and explain the relationship between angles of declination, maximum solar altitude and latitude (C) For the first few days of the voyage maximum solar altitudes (at mid-day) are tabulated below. Calculate the missing entries in the table. (D) In the workbook MathHomework.xls (see the first spreadsheet) edit and correct the formula for latitude in column E (labelled “wrong”).

Table 1: Sextant measurements of solar altitude

| Day | Month | Day Number | Altitude, ° | Declination ° | Latitude ° |
|-----|-------|------------|-------------|---------------|------------|
| 14 | 8 | 226 | 51.8 | 14.2 | -24.0 |
| 15 | 8 | 227 | 51.1 | 13.9 | ? |
| 16 | 8 | 228 | 50.3 | 13.5 | ? |
| 17 | 8 | 229 | 50.0 | 13.2 | ? |
| 18 | 8 | 230 | 49.5 | ? | ? |
| ... | ... | ... | ... | ... | ... |

Tips and hints

Part A continues on page 16. For part B, when working south of the Tropic of Capricorn select the north-direction button. The two sliders are very sensitive.

Determining Latitude from Maximum Solar Altitude

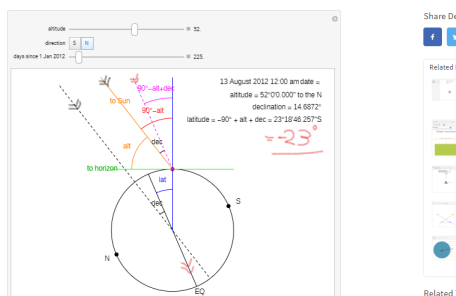


Figure 6: Screen capture from Wolfram solar demonstration. Make sure you select the North direction for Southern hemisphere. Chevrons indicate parallel lines. Note that the dashed lines are construction lines, parallel to the sun’s rays (black) or a line passing through the equatorial plane (purple).

For part (C) We have calculated nearly all declination angles, δ , for you. (We leave one blank space so

you can practice trigonometry functions.) It is subtended between the sun's rays and a line passing through the equatorial plane.^{4 5} . The declination angle follows

$$\delta = -23.45 \cos\left(\frac{360^\circ(d + 10)}{365}\right)$$

where d is the day number, $d=1$ on New Year's day. The latitude, λ , follows from

$$\lambda = A + \delta - 90^\circ$$

where A is the altitude. (Notice how the latitude can be expressed as -24° or $24^\circ S$)

For part (D) tips on using spreadsheets are at <https://exceljet.net/excel-formulas-and-functions> and <https://edu.gcfglobal.org/en/excelformulas/relative-and-absolute-cell-references/1/>. If you do not have Microsoft Office installed on your computer then there are free options, see for example <https://www.techrepublic.com/article/5-free-alternatives-to-microsoft-excel/>. I use Google Sheets, Libre Office and WPS; these should be OK for this Summer School.

In MathHomework.xls you will see that the wrong formula for latitude in cell E4 is
`=180+C4-B4`

You must correct this and copy the correction into other cells using a *relative* copy.

⁴<https://susdesign.com/popups/sunangle/declination.php>

⁵Scroll down to the animation at <https://www.pveducation.org/pvc/drom/properties-of-sunlight/declination-angle>

A: Multiple Choice

Record in your log book or diary the responses to the following. We shall discuss responses next week.

I wish to know the height of a tree, but cannot climb it. I stand 10 metres from the tree's base and measure the angle of elevation of an virtual line from my toes to the top of the tree. The angle is 60° . Is the height.

- 10 m
 - $10 \times \tan(60^\circ)m$
 - $10 \times \sin(60^\circ)m$
 - $\frac{10}{\sin(60^\circ)}m$
-

A can of soup is $80^\circ C$ hotter than its surroundings. It is left to cool, and after five minutes is $29.76^\circ C$ hotter than its surroundings. After a further five minutes it is $11.07^\circ C$ hotter than its surrounding. Yet another five minutes elapses - what then is the temperature difference?

- $80 \times 0.372 \times 0.372 \times 0.372^\circ C$
 - $10 \times 0.372 \times 0.372^\circ C$
 - $0^\circ C$
 - none of these
-

Which of the following statements is correct concerning the previous problem?

- The temperature difference decays exponentially, following Newton's law of cooling.
 - We use a quadratic formula to get the temperature difference.
 - Every five minutes, we divide the temperature difference by Euler's number
-

With regards to Figure 7 which of the following statements are true?

- a $\beta + \theta = \pi/2$
 - b $\sin(\beta) = \cos\theta$ and $\cos(\beta) = \sin(\theta)$
 - c for a distance s from the origin and along the red arrow, $\sin(\theta) = y/s$
 - d all the above
-

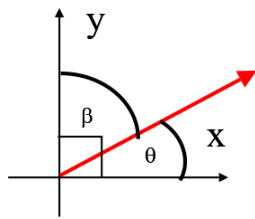


Figure 7: Sketch for multiple choice question.